**COVER LETTER**

25 November 2022

Prof. Dr. Daniel Ames

Editor in Chief

Environmental Modelling and Software (EMS)

Dear Dr. Ames, and the Editorial Board of EMS,

I am pleased to submit our manuscript entitled, “Inversion and forward estimation with process-based models: an investigation into cost functions, uncertainty-based weights and model-data fusion”, for consideration as an *original research article*.

*Overview:*

This study demonstrates methodological improvements in the inversion and forward estimation modelling processes when using limited, uncertain and spatial data. We test these using a case study of the Tephra2 model (ver 2.0), and field data from the eruption of the Kelud volcano in Indonesia in 2014. Our methodological contributions come in three components: (1) the selection of appropriate cost functions accounting for their behaviour and their associated implied distribution of residuals, (2) the treatment of differential uncertainty when combining multiple data sets, and (3) the leveraging of both forward model and data when estimating the spatial distribution of output.

*Importance:*

In the current model version of Tephra2, all data are treated equally in the optimisation step, differential uncertainty across datasets is not accounted for, the choice of cost function has no guidance, and the model outputs don’t harness the spatial characteristics in data. Such issues are common in earth, environment and hazard modelling. This study places attention on these issues, which receive little attention in modelling applications like tephra fallout modelling, but have a significant impact on the results. We believe that the study can benefit researchers interested in improving their estimates when conducting inversion and estimation of spatially-distributed data. The approach to select the cost function, treat uncertain data, and generate spatial estimates can also be applied to other earth, environment, and hazard models.

*Disclosure:*

We confirm that the manuscript has not been published or under consideration in another journal. We think this paper will make a good contribution of interest to the EMS readership community.

Sincerely,

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